Test 2A - MTH 2010

Dr. Graham-Squire, Spring 2015

2:09

Name: Key

⇒ total of 24

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Calculators, cell phones and computers are <u>not</u> allowed on this test.
4. Make sure you sign the pledge.
5. Number of questions = 14. Total Points = 70.

1. (5 points)

(a) Solve the subtraction problem: \$10.00 – \$3.99

$$\begin{array}{r} {}^{9}\not{1}\,{}^{10}\not{0}\,{}^{10} \\ \not{10}.\not{0}\not{0} \\ -\ 3.99 \\ \hline \$6.01 \end{array}$$

(b) Calculate $5 \times 10^4 + 0.324 \times 10^2$

$= 50{,}000 + 32.4 = 50{,}032.4$

(c) Multiply: $3\frac{1}{4} \times 2\frac{3}{5}$

$= \frac{13}{4} \times \frac{13}{5} = \frac{13 \times 13}{4 \times 5} = \frac{169}{20} = 8\frac{9}{20}$

(d) (2 points) Explain how to use a "mental math" strategy to calculate 9×99.

Write 99 as $100 - 1$ to get

$$\begin{aligned} 9 \times 99 &= 9 \times (100 - 1) \\ &= 9 \times 100 - 9 \times 1 \\ &= 900 - 9 \\ &= 891 \end{aligned}$$

2. (5 points) Which of the following problems could be solved by using the expression $4.5 \div 0.5$? Show your work and/or explain your answer to receive full credit!

$4.5 - 0.5$ ✗

(A) A concert last four and a half hours and you missed a half hour. How much of the concert did you see?

(B) The base of a triangle is 0.5 cm and the height is 4.5 cm. What is the triangle's area?

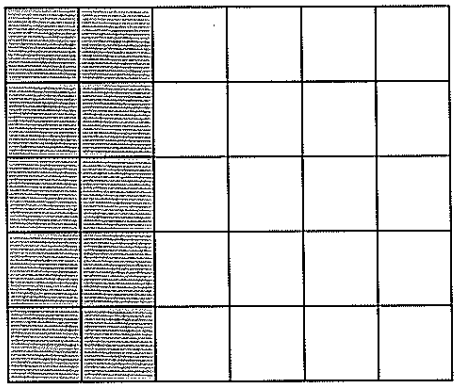
$\frac{1}{2} \cdot (0.5) \cdot 4.5$ ✗

(C) The length of a wooden plank is four and a half feet and you need pieces that are half a foot long. How many pieces can you get from the plank? Yes! This would be 9 pieces $4.5 \div 0.5$

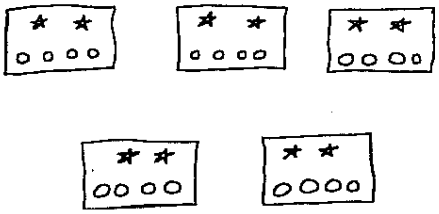
(D) Billy's house is 5 miles from Jenny's. If Billy can run 4.5 miles per hour, how long will it take him to reach Jenny's house?

$5 \div 4.5$ ✗

3. (5 points) Use one of the following diagrams to explain why $5 \times (2+4) = 5 \times 2 + 5 \times 4$, then generalize your argument to explain why the distributive property is true for *any* numbers, not just 5, 2 and 4.



OR



Shaded box is 5 groups of 2 = 5×2 boxes (rows)

Unshaded is 5 groups of 4 = 5×4 " ✓✓

Total boxes is 5×2 + 5×4

or, there are 5 groups of 2+4 boxes, since there are 2+4 boxes in each row. So total # of boxes is 5×(2+4). ✓✓

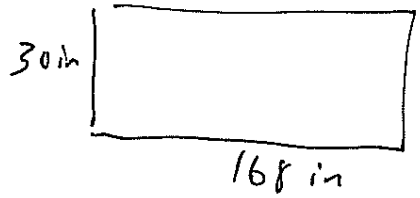
Thus 5×2 + 5×4 = 5×(2+4) since they both give the same # of boxes in the diagram.

In general A×(B+C) = A×B + A×C is true for all #s b/c you can draw a diagram with A rows, ✓ the B shaded columns and C unshaded columns, and repeat the argument above.

−0.5 if just do an example

4. (5 points) Your class is trying to approximate how many blades of grass are in a 30-inch by 14 foot rectangular plot of land that is covered with grass. The students approximate that there are an average of 26 blades of grass in one square inch of the plot. Use that to approximate how many blades of grass there are in the total 30-inch by 14-foot plot of land.

Need to know # of in^2 in 30-inch by 14-foot plot. 14 feet = 14 × 12 inches = 168 inches. ✓



$= 30 \times 168\ in^2$ ✓

$= 5040\ in^2$ ✓

```
  168
 ×30
 ---
  000
 240
1800
+3000
-----
 5040
```

Now do $5040 \times 26 =$ ✓

```
  5040
    26
------
  1 8 6
   240
     0
 30000
     0
   800
     0
+100000
-------
 131040
```

131,040 blades of grass ✓

5. (5 points) Adam pours $\frac{7}{8}$ of a cubic foot of gravel into an empty box. He then scoops out $\frac{1}{5}$ of the gravel in the box. How much gravel is left in the box?

   Which of the following expressions represent a means of finding a solution to this problem? Show your work and/or explain your answer to receive full credit!

   (A) $\frac{7}{8} - \frac{1}{5}$

   (B) $\frac{7}{8} \times \frac{1}{5}$

   (C) Some other expression involving fractions (if you choose this answer, you should indicate what the correct expression should be)

   (D) A solution cannot be calculated (if you choose this answer, you should indicate why a solution cannot be calculated)

$\frac{1}{5}$ of gravel in box is $\frac{1}{5}$ of $\frac{7}{8}$ of a cubic foot

$\Rightarrow \frac{1}{5} \times \frac{7}{8}$ is taken out, left with

$\frac{7}{8} - \left(\frac{1}{5} \times \frac{7}{8}\right)$ ft^3

6. (5 points) In the Kavango, it is tradition to tip your bartender with an amount that is approximately one-fourth of what you purchase (before sales tax is added). If you buy drinks costing \$29.42, after the 6% tax is added the total is \$31.19. Explain how you can use the tax to quickly approximate how much tip you should give. You should give an *approximation*, not the exact amount!

Tax was $31.19 − 29.42 = \$1.77$

$1.77 \times 4 = 7.08$

So 6% ⟷ 1.77

×4 ↓ ↓ ×4

24% ⟷ \$7.08 ✓✓✓ for answer

So tip should be a little over \$7, maybe \$7.25.

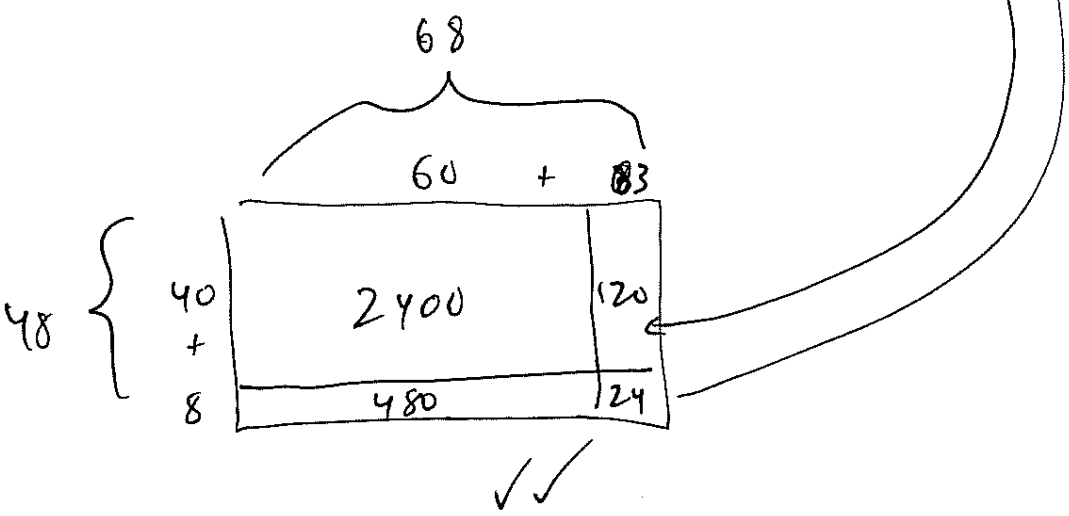
✓✓ for using tax

7. (5 points) Multiply 63×48 using whatever multiplication algorithm you choose. Then use an array or the distributive property to explain <u>why</u> your algorithm gives the correct answer to the problem.

$$\begin{array}{r} 63 \\ \times\ 48 \\ \hline 24 \\ 480 \\ 120 \\ 2400 \\ \hline 3024 \end{array}$$

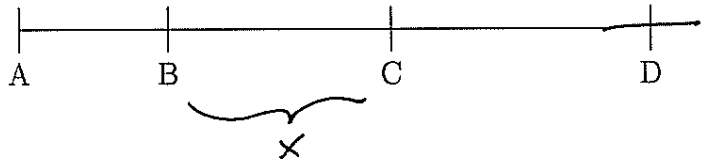
✓

Boxes in the array correspond to parts in the partial products algorithm. ✓



✓✓

8. (5 points) In the figure below, the distance from B to C is twice the distance from A to B, and the distance from C to D is equal to half the distance from A to C. If the distance from B to C is x, what is the distance from A to D? Show your work and/or explain your answer to receive full credit! Note: the diagram may not be completely to scale.



(A) $\left(\frac{1}{2}\right)x$

(B) $\left(\frac{3}{4}\right)x$

(C) $\left(2\frac{1}{4}\right)x$

(D) $\left(2\frac{1}{2}\right)x$

2.09
2.23
14

$BC = 2 \cdot AB$

$x = 2 \cdot AB$

$\frac{x}{2} = AB$

so A to C is

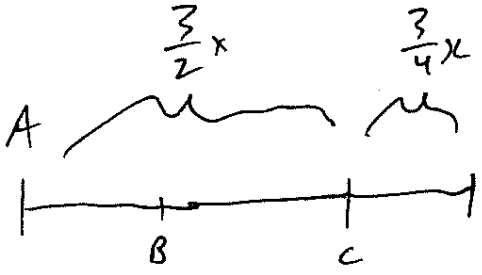
$x + \frac{x}{2} = \frac{3}{2}x$

C to D $= \frac{1}{2}(AC)$

$\Rightarrow$ C to D $= \frac{1}{2}\left(\frac{3}{2}x\right)$

$= \frac{3}{4}x$

3 for work, ✓✓✓
2 for answer ✓✓



$\frac{3}{2}x + \frac{3}{4}x = \left(\frac{3}{2} + \frac{3}{4}\right)x$

$= \left(\frac{6}{4} + \frac{3}{4}\right)x$

$= \frac{9}{4}x$

$= \left(2\frac{1}{4}\right)x$

9. (5 points) Here is a method that a student used for subtraction: 2:27

$$\begin{array}{r} \overset{1}{\cancel{7}}\,\overset{15}{\cancel{5}}\,8 \\ -\ 3\,\cancel{2}\ 8\ 6 \\ \hline 4\ 7\ 2 \end{array}$$

Which of the following is correct?

(A) The student used a method that worked for this problem and can be generalized to any subtraction problem.

(B) The student used a method that worked for this problem and that will work for any subtraction problem that only requires one regrouping; it will not work if more regrouping is required.

(C) The student used a method that worked for this problem and will work for all three-digit subtraction problems, but will not work for larger problems.

(D) The student used a method that does not work. The student made two mistakes that cancelled each other out and was lucky to get the right answer for this problem.

Instead of subtracting 1 from 7 when they borrow, they add 1 to the 2 (the # below) since $7-1-2$ is same as $7-(2+1)$, this will always be legitimate.

2:29
− 2124

10. (5 points) A student cannot remember how to multiply decimal numbers such as 1.24×8.7, but they can calculate that $124 \times 87 = 10788$.

3 (a) Using either a "multiply by tens, then divide by tens" or a "fraction" argument, explain to the student why the standard method for multiplying decimal numbers works.

(2)

2:43

2 (b) Explain to the student how to use approximation to figure out where to place the decimal in your answer.

(a) $1.24 \longrightarrow \times 10 \times 10 \longrightarrow 124$

$\times 8.7 \longrightarrow \times 10 \longrightarrow \times 87$

10788

$\div 10 \leftarrow$

$10.788 = \div 10 \leftarrow \div 10 \leftarrow$

have to undo what was done with multiplication above.

(b) $1.24 \approx 1$

$8.7 \approx 9$ so $1.24 \times 8.7 \approx 1 \times 9 = 9$

~~10788~~

depending on decimal placement, could be

0.10788

1.0788

10.788 → closest to 9 ✓

107.88

1078.8

11. (5 points) One astronomical unit (AU) is approximately 9×10^7 miles. Earth is approximately 2.7 trillion miles from the outer reaches of the Milky Way galaxy. Using the approximations above, how many AU is it from Earth to the outer reaches of the Milky Way galaxy? Write your answer in scientific notation.

2.7 trillion is 2,700,000,000,000 ~~0~~ 0.5

$= 2.7 \times 10^{12}$

$\frac{2.7 \times 10^{12}}{9 \times 10^7}$ ~~0~~ 0.5 ✓

$= \frac{2.7}{9} \times \frac{10^{12}}{10^7}$ ✓ ✓

$= 0.3 \times 10^5$

$= 30000$

$= 30,000$

$= \boxed{3 \times 10^4}$

✓

12. (5 points) A crab scuttles at a pace of 70 feet per minute. You ask your students to calculate how long it will take the crab to cross the length of a football field (300 feet). One student quickly does the calculation and says "it will take the crab 4 minutes and 20 seconds"! Does the student have the correct answer? If so, explain how the student came to their answer. If not, explain what is wrong with the student's calculation, and use their work to explain to them how to do it correctly–you do not have to calculate the exact answer, just explain what the correct calculation would be.

Student is wrong ✓

Calculation is $\frac{300}{70} = \frac{30}{7} = 4\frac{2}{7}$

or

4 remainder 2

Student did $\frac{300}{70} = 4$ remainde 20

✓✓✓ but the 20 is 20 feet left in the field to cross, not 20 seconds.

The correct answe is $4\frac{2}{7}$ of a minute. ✓

13. (5 points) Explain in two different ways why division by zero is undefined. It is fine to use an example problem like $4 \div 0$.

2.5 (a) Use a word problem to illustrate how division by zero does not make sense.

2.5 (b) Use the fact that division is the same as multiplication by an unknown to explain why division by zero is not defined.

(a) Bob has four cookies. He puts 0 in each box. How many boxes does he need?

$\Rightarrow$ does not make sense b/c putting 0 in the box will never get up to 4.

(b) $4 \div 0 = \square$

$\Rightarrow 0 \times \square = 4$

There is no # that can work here b/c 0 × anything = 0, not 4.

+0.5 for attempting

+1 for saying that 0 times anything is zero.

14. (5 points) If n is a positive number, which of the following could be *negative*? Show your work and/or explain your answer to receive full credit!

2:50
2:43
7

Smallest positive # is 1.

(A) $(n+1)(n-1)$ ⇒ could be zero, but not negative.

(B) $(n+1)(n+2)$ ⇒ always positive

(C) $n(n-2)$ ⇒ if $n=1$ get $1\cdot(1-2)=-1$

(D) $n(n)$ ⇒ always positive.

Extra Credit(2 points) A student solves a mixed-fraction calculation as follows:

$$5\frac{1}{6}-2\frac{5}{6}=2\frac{6}{6}$$

Where did the student go wrong, and how would you explain to them how to fix it? Try to use the student's thought process as much as possible.

Student probably carried a 10 from 5, but should have only carried 6.

$$\begin{array}{r} 4\,\not{5}^{\,11}\,\tfrac{x}{6} \\ -\;2\;\tfrac{5}{6} \\ \hline 2\;\tfrac{6}{6} \end{array}$$

Correctly would be

$$\begin{array}{r} 4\,\not{5}^{\,7}\,\tfrac{1}{6} \\ -\;2\;\tfrac{5}{6} \\ \hline \boxed{2\;\tfrac{2}{6}} \end{array} \quad \text{or } 2\tfrac{1}{3}$$